Catalan Numbers

- Denoted by C_n.

- $$C_n = \frac{\binom{2n}{n}}{n+1} = \frac{(2n)!}{(n)!\,(n)!\,(n+1)}$$

- $$C_{n+1} = \frac{2(2n+1)}{n+2} \cdot C_n$$

Proof:

$$C_{n+1} = \frac{\binom{2(n+1)}{n+1}}{n+2} = \frac{(2n+2)!}{(n+1)!\,(n+1)!\,(n+2)}$$

$$\frac{C_{n+1}}{C_n} = \frac{(2n+2)!}{(n+1)!\,(n+1)!\,(n+2)} \cdot \frac{(n)!\,(n)!\,(n+1)}{(2n)!}$$

$$= \frac{(2n+2)(2n+1)(2n)!}{(n+1)!\,(n+1)!\,(n+2)} \cdot \frac{(n)!\,(n)!\,(n+1)}{(2n)!}$$

$$= \frac{2\cancel{(n+1)}(2n+1)\cancel{(2n)!}}{\cancel{(n+1)}\cancel{(n)!}\,\cancel{(n+1)}\cancel{(n)!}\,(n+2)} \cdot \frac{\cancel{(n)!}\,\cancel{(n)!}\,\cancel{(n+1)}}{\cancel{(2n)!}}$$

$$= \frac{2(2n+1)}{n+2}$$

$$\therefore\ C_{n+1} = \frac{2(2n+1)}{n+2} \cdot C_n$$

- Catalan numbers also satisfy the recurrence relation
  $C_{n+1} = C_0 \cdot C_n + C_1 \cdot C_{n-1} + \ldots + C_{n-1} \cdot C_1 + C_n \cdot C_0$

- $C_0 = 1$